Drag formula :

If we impose equal and opposite velocity to the sphere and the liquid together. We find the motion of the sphere of radius 'a' with velocity U through the liquid rest at infinity. Mathematically, the solution to the motion of sphere with velocity U through fluid at rest is obtained. If we remove the 1st term in the expression of ψ

Hence $\psi = \frac{3}{4} UaR \left[1 - \frac{1}{3}\frac{a^2}{R^2}\right] \sin^2\phi \leftarrow \boxed{\text{Drop the 1st term}}$

$$\therefore q_R = -\frac{3}{2} U\cos\phi \left[\frac{a}{R} - \frac{1}{3}\frac{a^3}{R^3}\right]$$

$$q_\phi = \frac{3}{4} U\sin\phi\left[\frac{a}{R} + \frac{1}{3}\frac{a^3}{R^3}\right]$$

If D is the Drag on the sphere, then ~~workdown~~ worked done by the Drag force

= Dissipation of energy

Now, the sphere is moving with velocity U

$\therefore$ Rate of work done by the drag = Dissipation of energy (= DU)

$$= \mu \iiint_V \vec{\xi}^2 \, dV \quad \left[\begin{array}{l}\text{The fluid is} \\ \text{at rest at} \\ \text{Infinity}\end{array}\right]$$

But $\vec{\xi}$ has only one component

$$\xi_0 = \frac{1}{R}\left[\frac{\partial}{\partial R}(Rq_\phi) - \frac{\partial}{\partial\phi}(q_R)\right]$$

$$= \frac{1}{R}\left[\frac{\partial}{\partial R}\left\{\frac{3}{4}U\sin\phi\left(a+\frac{1}{3}\frac{a^3}{R^3}\right)\right\} - \frac{\partial}{\partial\phi}\left\{-\frac{3}{2}U\cos\phi\left(\frac{a}{R}-\frac{1}{3}\frac{a^3}{R^3}\right)\right\}\right]$$

$$= \frac{1}{R}\left[\frac{3}{4}U\sin\phi\left(-\frac{1}{3}\frac{2a^3}{R^3}\right) - \left\{\frac{3}{2}U\sin\phi\left(\frac{a}{R}-\frac{1}{3}\frac{a^3}{R^3}\right)\right\}\right]$$

$$= \frac{1}{R}\left[-\frac{1}{2}\frac{a^3}{R^3} - \frac{3a}{2R} + \frac{1}{2}\frac{a^3}{R^3}\right]U\sin\phi$$

$$= -\frac{3aU\sin\phi}{2R^2}$$

$$DU = \mu\int_{R=a}^{\infty}\int_{\phi=0}^{\pi}\int_{\lambda=0}^{2\pi}\left[\frac{9a^2U^2\sin^2\phi}{4R^2}\cdot R^2\sin\phi\right]dR\,d\phi\,d\lambda$$

$$= 2\pi\mu\int_{R=a}^{\infty}\int_{\phi=0}^{\pi}\frac{9a^2U^2\sin^2\phi}{4R^4}\cdot R^2\sin\phi\,d\phi\,dR$$

$$= 2\pi\mu\frac{9a^2U^2}{4}\int_{R=a}^{\infty}\frac{dR}{R^2}\int_{\phi=0}^{\pi}\sin^3\phi\,d\phi$$

$$= \frac{9a^2\pi\mu U^2}{2}\left[-\frac{1}{R}\right]_a^{\infty}\int_{\phi=0}^{\pi}\sin^3\phi\,d\phi$$

$$= \frac{9aU^2}{2}\mu\pi\frac{4}{3} = 6\pi\mu aU^2$$

$$\Rightarrow D = 6\pi\mu aU.$$

This is known as Drag formula of Stokes.

This is the well known stoke's formula for the drag of a sphere.

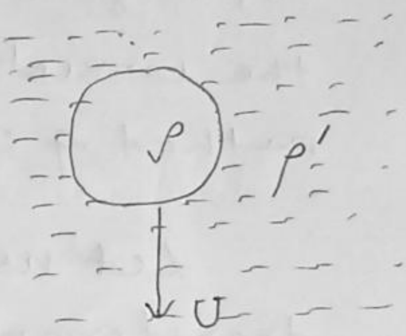
We may use this form of drag to find the terminal velocity of a sphere when it moves through the fluid vertically down ward under gravity.

If U be the velocity, ρ' be the density of the sphere, ρ be the density of the fluid outside, then

total force on the sphere = its weight − bouyancy

$$= \frac{4\pi}{3}\left(\rho' - \rho\right) a^3 g$$

and $\frac{4\pi}{3}\left(\rho' - \rho\right) a^3 g = 6\pi\mu a U$



$$\Rightarrow U = \frac{2}{9\mu}\left(\rho' - \rho\right) a^2 g$$