Magnetization
Lecture 3

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Larmor Precession:

The precession of an orbit of an electron under the influence of applied magnetic field is called Larmor Precession. As shown in $Fig\ 1$ let us take any arbitrary direction or the angular momenta M relative to the magnetic field H.

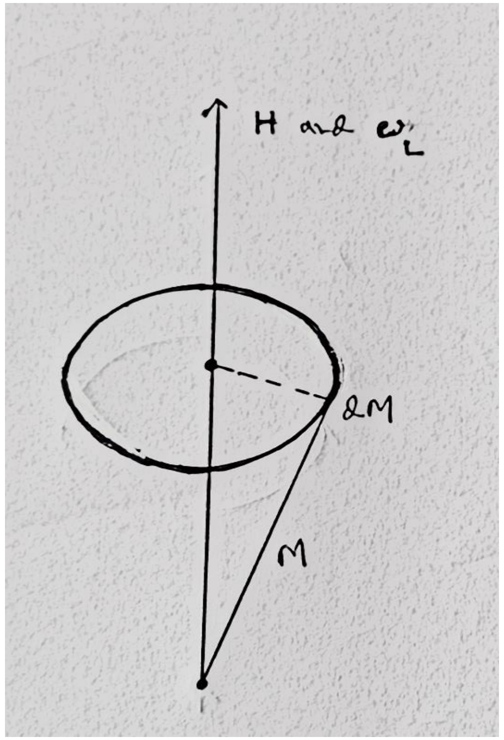

Fig 1

Then the magnetic dipole moment will be

$$m_d = -\left(\frac{e}{2mc}\right) M \rightarrow (i)$$

Hence the torque on the dipole

$$\tau = m_d \times H = -\left(\frac{e}{2mc}\right) M \times H \rightarrow (ii)$$

The equation (ii) represents the equation of motion of M having angular velocity ω_L precessing about H, where

$$\omega_L = \left(\frac{e}{2mc}\right) H \rightarrow (iii)$$

Here ω_L is known as Larmor Precession. Now we know that

$$\left(\frac{e}{2mc}\right) = 1.40 \times 10^6 \ /\sec \ /gauss$$

But even for a field having intensity 10^6 gauss, Larmor frequency ω_L is much smaller than the angular frequency of the electron. The deviation is due to the assumption that M is independent of H. In other words in the above expression we have assumed that the orbit remains unchanged under the influence of the magnetic field.

Paramagnetic Susceptibility (Classical Theory) :

Consider a medium having N magnetic dipole per unit volume. Let us assume that the interaction between the dipole is weak so that each dipole is subjected to a magnetic field H and the dipoles are assumed to be rotating freely.

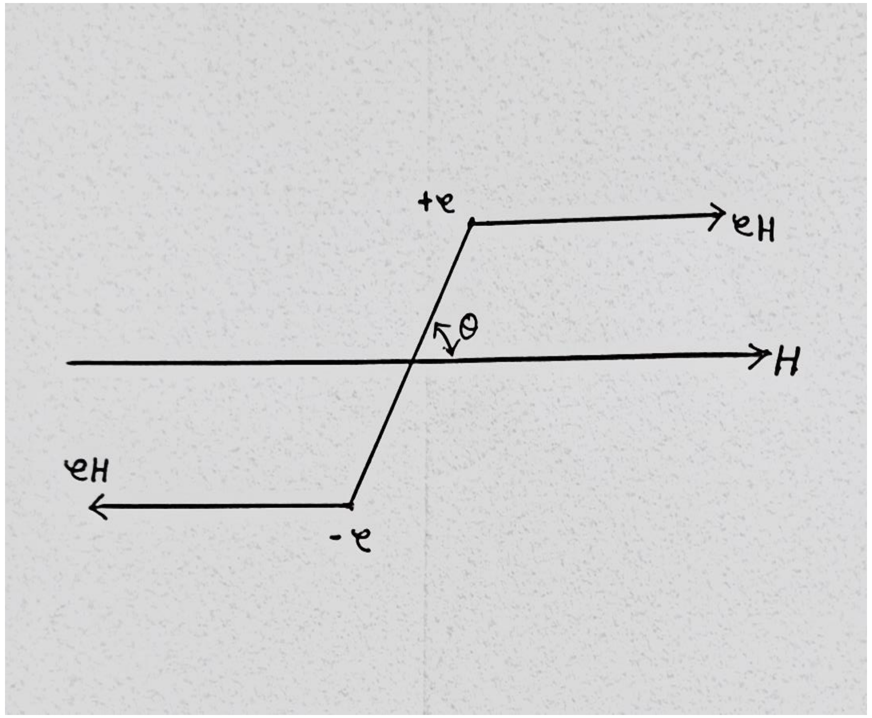

Fig 2

In the absence of magnetic field H the orientation of the dipoles are random. As a result the medium has no resultant dipole moment. Now the application of field H will produce a torque on each dipole and dipole wants to rotate in the direction of field as shown in Fig 2. Here we have taken the potential energy to be zero for a dipole making an angle $\frac{\pi}{2}$ with the field H. Hence potential energy for the dipole making an angle θ with H will be $(-m_d H cos\theta)$. Therefore according to the statistical mechanics the probability for a dipole making an angle between $\theta \ and \ \theta + d\theta$ with H is proportional to

$$2\pi sin\theta d\theta e^{(\mu H cos\theta / KT)}.$$

The average component of dipole moment along the field will be

$$m_d\langle cos\theta\rangle = \frac{\int_0^{\pi} m_d cos\theta 2\pi sin\theta d\theta e^{(\mu H cos\theta/KT)}}{\int_0^{\pi} 2\pi sin\theta d\theta e^{(\mu H cos\theta/KT)}}$$

Let us put $\frac{\mu H cos\theta}{KT} = x$ and $\frac{\mu H}{KT} = a$

Therefore

$$\langle cos\theta\rangle = \frac{1}{a} . \frac{\int_{-a}^{a} x e^x dx}{\int_{-a}^{a} e^x dx} = \frac{e^a + e^{-a}}{e^a - e^{-a}} - \frac{1}{a} = L(a)$$

Where $L(a)$ is called $Langevin\ Function$. As shown in $Fig\ 3$, $L(a)$ is plotted against a. One can conclude from the figure that for the large value of a the function $L(a)$ approaches to a saturated value . This situation corresponds to the complete alignment of the dipole along the field direction.

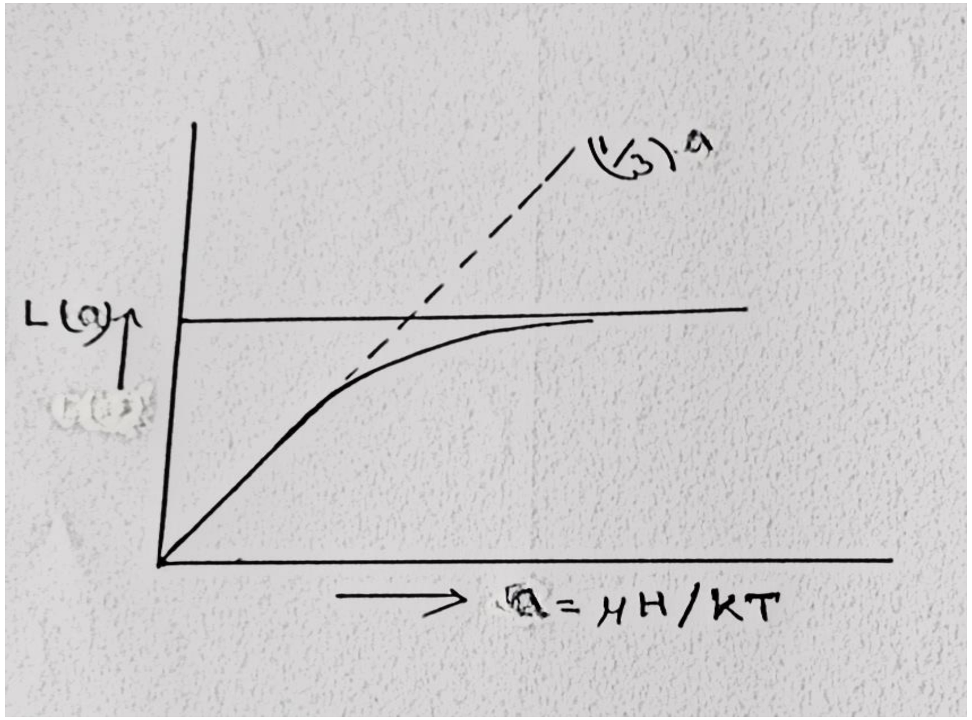

Fig 3

When the intensity of the field and the temperature are not too high we take the approximation

$$a \ll 1 \ or \ \frac{\mu H}{KT} \ll 1$$

i.e. $\mu H \ll KT$

In this case $L(a) = \frac{a}{3}$

Therefore

$$m_d \langle cos\theta \rangle = \frac{m_d^2}{3KT} \ H$$

Since the medium contains N dipole per unit volume we have

$$M = N\frac{m_d^2 H}{3KT}$$

$$\frac{M}{H} = \chi = \frac{Nm_d^2}{3KT}$$

The susceptibility of paramagnetic substance depends on temperature. At room temperature this expression is satisfied except for a very low temperature. From this expression we can say that

$$\chi = \frac{Constant}{T}$$

Paramagnetic Susceptibility (Quantum Theory) :

Consider a medium having N atoms per unit volume subjected to an external field of intensity H . According to quantum theory the paramagnetic moment of an atom is restricted to a finite set of orientation. Suppose the total angular momentum quantum number of each atom is J. Corresponding possible components of the magnetic moment will be $M_J g\mu_B$, where M_J is the magnetic quantum number and μ_B is the *Bohr magnetron.* Hence the potential energy of a dipole along the applied field H will be $-M_J g\mu_B H$. Therefore the magnetization is

$$M = N\frac{\sum_{-J}^{+J} g\mu_B exp\left(\frac{M_J g\mu_B H}{KT}\right)}{\sum_{-J}^{J} exp\left(\frac{M_J g\mu_B H}{KT}\right)}$$

Where M_J have values $+J, (J+1), \dots \dots . -(J-1),\ -J$

The above expression is obtained from Maxwell-Boltzmann Statistics.

Condition A:

Now suppose $\frac{M_J g \mu_B H}{KT} \ll 1,$ under this condition the exponential term of the above expression reduces to $\left(M_J g \mu_B H / KT\right)$.

Again $\sum M_J g \mu_B = 0$, because

$$\sum_{-J}^{J} M_J = J + (J+1) + \cdots . -(J-1) - J = 0$$

And

$$\sum_{-J}^{+J} M_J^2 = J^2 + (J+1)^2 + \cdots \{-(J-1)\}^2 + (-J)^2$$

$$\sum_{-J}^{+J} M_J^2 = \frac{2J(J+1)\,(2J+1)}{6}$$

Therefore we can write

$$M = \frac{N g^2 \mu_B^2\, J\,(J+1)\,(2J+1)}{3KT(2J+1)} H$$

$$M = \frac{NJ(J+1)g^2\mu_B^2}{3KT}H$$

The total magnetic moment μ_J corresponding to the total angular momentum quantum number J will be

$$\mu_J = g\sqrt{J(J+1)}\mu_B$$

$$\mu_J^2 = J(J+1)g^2\mu_J^2$$

$$M = \frac{N\mu_J^2 H}{3KT}$$

Therefore susceptibility

$$\chi = \frac{M}{H} = N\ \frac{\mu_J^2}{3KT}$$

At low temperature and strong magnetic field the condition A is not valid. After simplification ones gets the following expression for magnetization is

$$M = NgJ\mu_B B_J(x)$$

Where $B_J(x)$ is the Brillouin function and $x = \frac{gJ\mu_B H}{KT}$.

Physically one gets saturation at low temperature.

Para-magnetism requires the existence of incompletely filled electronic shell. Thus transition group elements are paramagnetic substances. Out of these the rare earth group have incomplete $4f$ shell and iron group have incomplete $3d$ shell shows the extensive paramagnetic effect.