Refraction through a spherical surface

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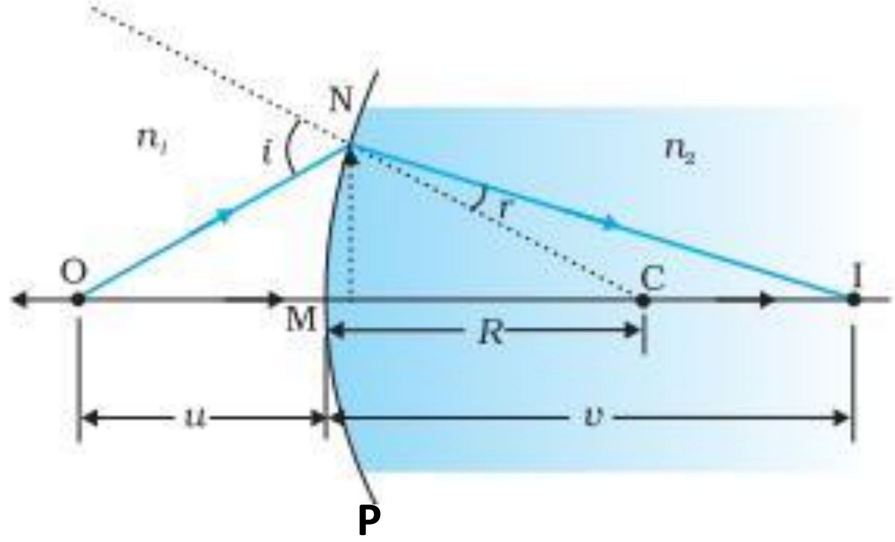
Refraction at a spherical interface:.

Let MNP be a spherical. The rays are incident from a medium of refractive index n_1 and refracted to another medium of refractive index n_2.

Assuming the aperture (or the lateral size) of the surface to be small compared to other distances involved, so that small angle approximation can be made.

Consider NM will be taken to be nearly equal to the length of the perpendicular from the point N on the principal axis.



Considering small angles,

tan<NOM = $\frac{MN}{OM}$

tan < NCM = $\frac{MN}{MC}$

tan<NIM = $\frac{MN}{MI}$

Now, for ΔNOC, <i is the exterior angle.

Therefore, i = ∠NOM + ∠NCM

$$i = \frac{MN}{OM} + \frac{MN}{MC} \text{ ---------------- (1)}$$

Similarly, r = ∠NCM – ∠NIM

$$r = \frac{MN}{MC} - \frac{MN}{MI} \text{ ----------------(2)}$$

Now, by Snell's law $\frac{n_2}{n_1} = \frac{sin\ i}{sin\ r}$

For small angles sin i =i and sin r =r

$$n_1 i = n_2 r$$

Substituting i and r from Equation. (1) and (2), we get

$$\frac{n_1}{OM} + \frac{n_2}{MI} = \frac{n_2 - n_1}{MC} \quad ----- (3)$$

Applying the Cartesian sign convention,
OM = –u, MI = +v, MC = +R
Substituting these in Eq. (3), we get,

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} \quad \text{--------(4)}$$

Equation (4) gives us a relation between object and image distance in terms of refractive index of the medium and the radius of curvature of the curved spherical surface. It holds for any curved spherical surface.

Lens maker formula:

We know the relation for refraction at spherical surface is

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} \quad \text{-----------(1)}$$

The symbols have their usual meaning.

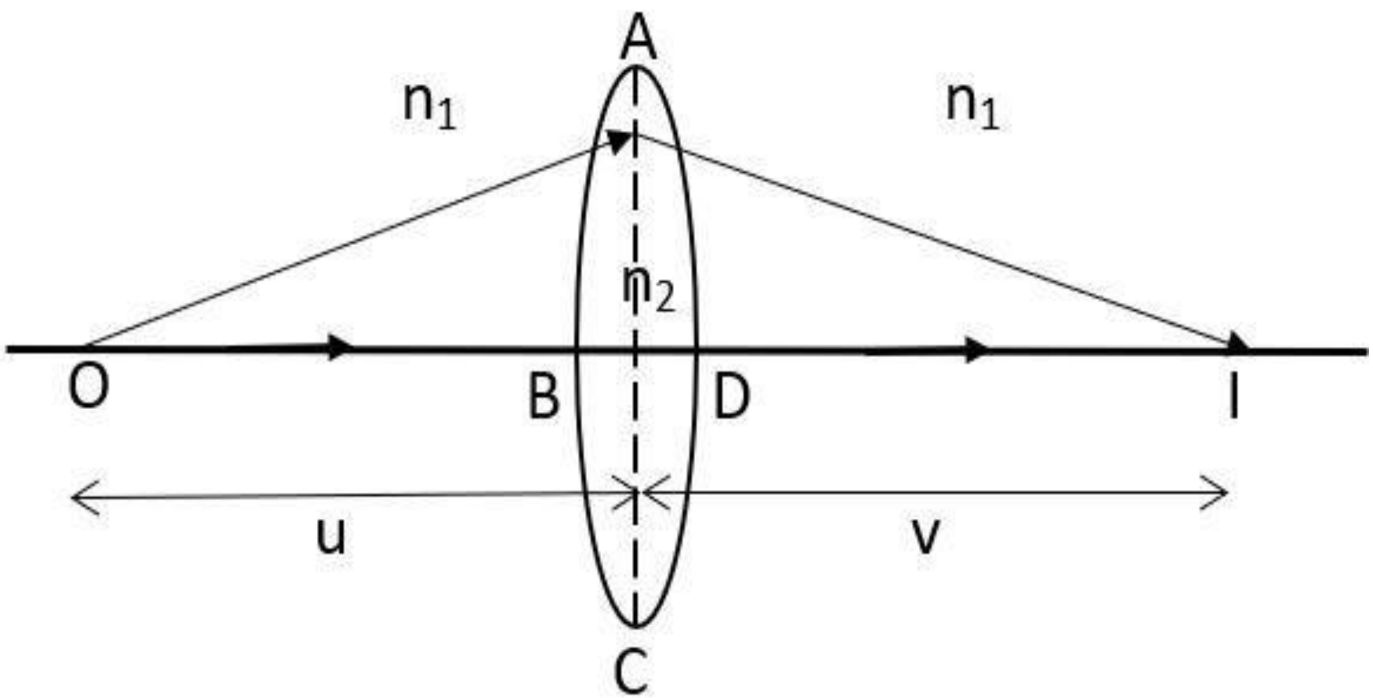
Let us assume a double convex lens with radius of curvature R.

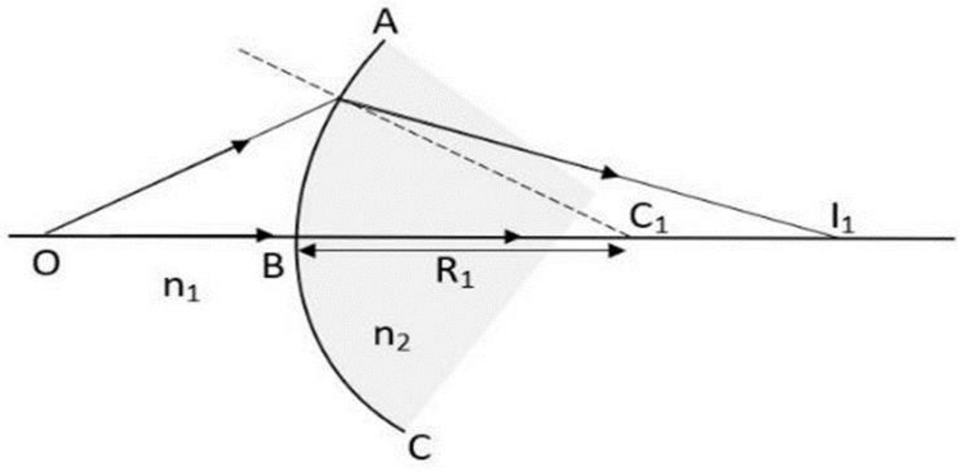
Where n_1 - r.i of the surrounding medium

n_2 - r. i of glasses

v - image distance and

u- object distance from the lens .


A
n_1
n_1
n_2
O
B
D
I
u
v
C


A
C_1
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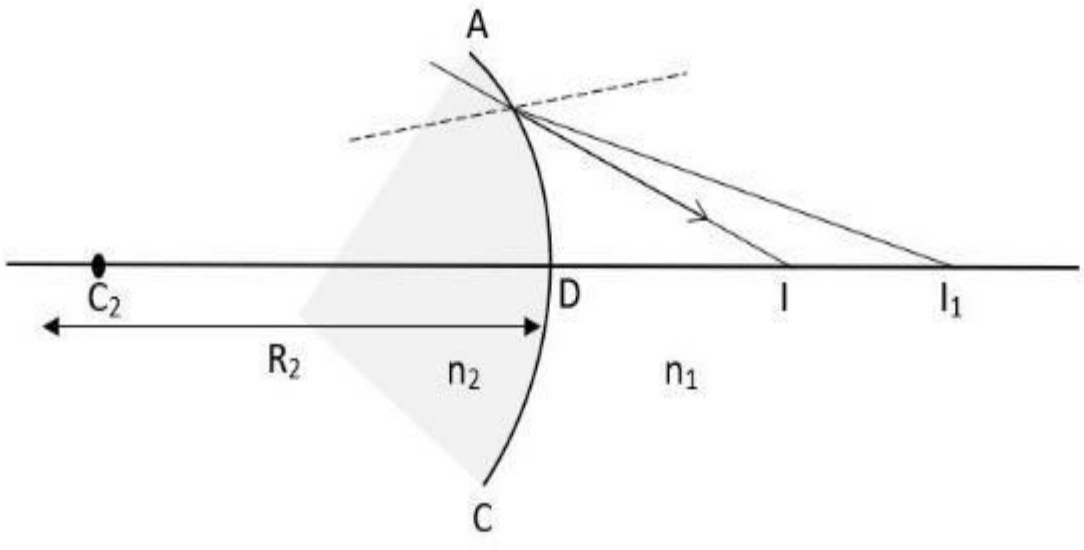

A
C_2
D
I
I_1
R_2
n_2
n_1
C

The refraction first takes place at the interface ABC, then the 1st image will form at I_1Type equation here. which will then serve as a virtual image for the second interface ADC.

Now, applying the above equation for surface ABC, we get

$$\frac{n_1}{OB}+\frac{n_2}{BI_1}=\frac{n_2-n_1}{BC_1} \text{....... (i)}$$

Similarly, we can write the equation for second interface as $\frac{n_2}{DI_1}+\frac{n_1}{DI}=\frac{n2-n1}{DC_2}$ -----(ii)

Here, the lens is thin, so $BI_1=DI_1$.

Adding the equations (i) and (ii), we get

$$\frac{n_1}{OB}+\frac{n_1}{DI}=\frac{n_2-n_1}{DC2} \text{ ----------(iii)}$$

If the object is assumed to be at infinity then $OB=\infty$

$$\therefore \quad \frac{n_1}{DI} = (n_2 - n_1)\left(\frac{1}{BC_1} + \frac{1}{DC_2}\right)$$

Dividing both sides by n_1, we get

$$\frac{1}{DI} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{BC1} + \frac{1}{DC2}\right)$$

As $DC_2 = -R_2$, $DI = f$ and $BC_1 = R_1$

Then we get $\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \text{ ------(IV)}$$

Hence, we get the Lens Maker's formula.

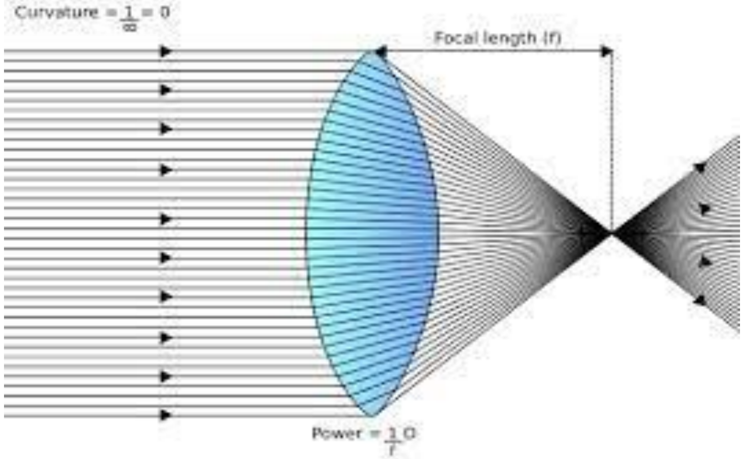
Power of a lens:

The ability of a lens to converge or diverse a light beam when it passer through a lens is called power of a lens.

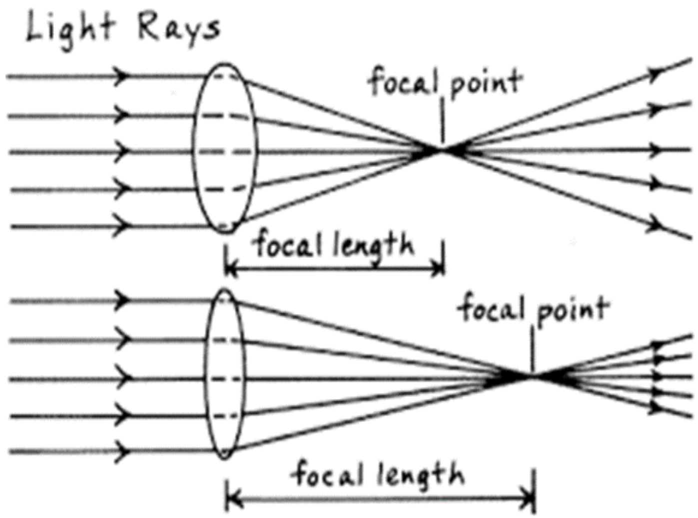
The power of a lens (P) is inversely proportional to its focal length

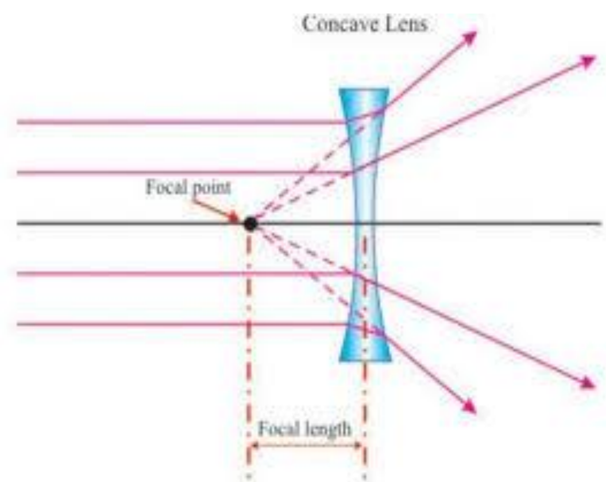
$$\therefore P \infty \frac{1}{f\ in\ meter}$$ f- focal length in meter

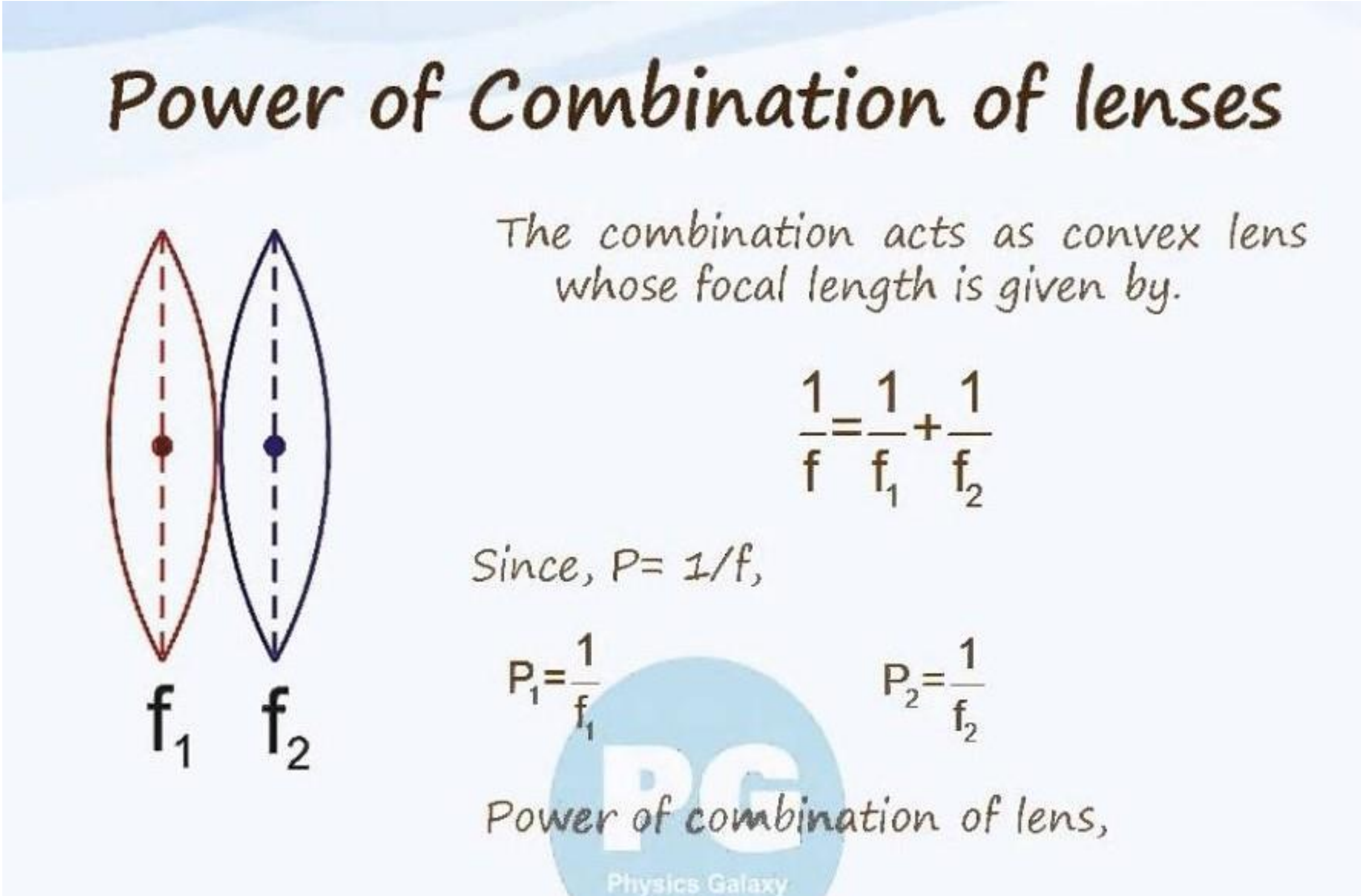
S.I unit of power = Dioptre (D)



Power of a convex lens is positive and of a concave lens is negative







So power of the combination of lenses $P = P_1 + P_2$