Boundary Layer Approximation: (BLT)

Equation of motion:

$$\sigma_{ij} + \rho b_i = \rho \dot{v}_i$$

cond^n for inviscid fluid $\longrightarrow$ Euler's eq^n of motion

cond^n for viscous fluid $\longrightarrow$ Navier-stoke's eq^n of motion

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q}\cdot\vec{\nabla})\vec{q} = \vec{F} - \frac{1}{\rho}\vec{\nabla}p + \nu \nabla^2 \vec{q}$$

Convective term
or
Inertia term

The equation of motion for incompressible viscous fluid (with no body forces like gravity) is given by

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q}\cdot\vec{\nabla})\vec{q} = -\frac{1}{\rho}\vec{\nabla}p + \nu \nabla^2 \vec{q} \longrightarrow (1)$$

where $\nu = \frac{\mu}{\rho}$ is called the Kinematic viscosity.

The exact solution of this equation is obtained only for some simple flow fluids. But for many other flow fluids, it is very difficulte to solve this equation. This difficulty is mostly due to the presence of non-linear term $(\vec{q}\cdot\vec{\nabla})\vec{q}$ in the expression.

We know that the Reynolds number R is defined by the ratio of the inertia term and the viscous term

i.e. $R = \frac{I.T.}{V.T} = \frac{UL}{\nu}$

$$\boxed{2300 < R < 2800}$$

Laminar (R < 2300) — turbulent (R > 2800)

where U is the representive (or characteristic) velocity, L is the representive (or characteristic) length and ν is the kinematic viscosity. Thus, the Reynolds no gives the relative importance of the inertia force w. r. to the viscous force. If $R << 1$, the inertia term is less important then the viscous term. If, on the otherhand, $R >> 1$, the inertia term is much more important than the viscous term (This is the case we are now concerned with).

In the case of large Reynolds no (i.e., large velocity or small viscosity). The viscous term may be neglected for an approximate of the solution of (1) and this approximation is used for investigation of the motion in the main body of the fluid.

This means, in the main body of the fluid, the motion can be investigated simply by following the perfect fluid theory.

But in the immediate neighbourhood of the solid boundaries, the approximation with the neglected of the viscous term is not permissible for the condition of no slip on the solid boundaries will be violated. If the condition of no-slip were not to be satisfied in the case of a real fluid, there would be no appreciable difference between the ~~fluid~~ field of flow of real fluid and that of perfect (or inviscid) fluid. In fact, the influence of viscosity at high Reynolds number is confined to a very thin layer in the immediate neighbourhood of the solid boundary or wall. In that thin layer, the velocity of a fluid increases from zero value at the wall to its full value which corresponds to external frictionless force. This layer under consideration is called the boundary layer and the concept is due to L. Prandtl (1904).

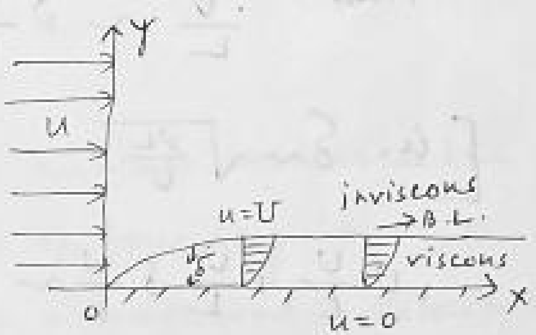
Hence, for the purpose of mathematical analysis, the fluid of flow in the case of small viscosity where solid wall is present can be divided into two regions.

(i) The thin boundary layer near the wall in which friction must be taken into account.

(ii) The region outside the boundary layer where the forces due to friction are small and may be neglected and where therefore the perfect fluid theory offers a very good approximation.

§: Some basic difinition (Boundary Layer thickness, Displacement thickness, Momentum thickness, Energy thickness, Drag and Lift)

(I). Boundary Layer thickness:



For the sake of simplicity, we consider a two-dimensional flow along a thin flat plate as shown in the figure and suppose that there is developed a laminer boundary layer thickness which has not separated and is estimated in the following ways:

Accross the boundary layer, the flow velocity changes from zero value at the plate to some finite value. Hence, if δ be the thickness of the boundary layer at some point on the plate, then the x-component of the velocity U changes from 0 to U over a distance δ, where U is the main stream velocity.

Now, the velocity gradient $\frac{\partial u}{\partial y}$ is of order of magnitude

(4)

$\frac{U}{\delta}$

i.e. $\frac{\partial u}{\partial y} \sim \frac{u}{\delta}$ and so $\frac{\partial^2 u}{\partial y^2} \sim \frac{u}{\delta^2}$

Again, for a plate of length L, the velocity gradient

$$\frac{\partial u}{\partial x} \sim \frac{U}{L}$$

Hence by equation of continuity,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \text{ , we have,}$$

$$\frac{\partial u}{\partial y} \sim \frac{\partial u}{\partial x} \sim \frac{U}{L} \quad ;$$

$$\frac{\partial v}{\partial y} = \frac{U}{L}$$

$$\int dv = \int \frac{U}{L}\, dy$$

$$\Rightarrow v = \frac{U}{L}\left[y\right]_0^{\delta}$$

Hence $v \sim \frac{U}{L}\delta$

Inside the boundary layer the motion is nearly parallel to x-axis.

Now, inside the boundary layer the inertia and viscous forces are of comparable order of magnitude.

Thus $\quad \frac{U^2}{L} \sim \nu \frac{U}{L^2}$

$$\text{I.T.} \quad (\vec{q}\cdot\vec{\nabla})\vec{q}$$

$$x \to u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$\text{VT} \to \nu \nabla^2 \vec{q}$$

$$x \to \nu \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

[i.e. $\delta \sim \sqrt{\frac{\nu L}{U}}$

$$U\frac{U}{L} + \frac{U}{L}\delta\frac{U}{\delta} \sim \nu\left(\frac{U}{L^2} + \frac{U}{\delta^2}\right)$$

i.e. $\quad U + U \sim \nu \frac{U}{\delta^2}$

i.e. $\quad 2U \sim \nu \frac{U}{\delta^2}$

i.e. $\quad U \sim \frac{u}{\delta^2}$]

$$\text{I.T.} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \qquad \text{V.T.} = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{U^2}{L} + \frac{U}{L}\delta\frac{U}{\delta} \sim \nu\left(\frac{U}{L^2} + \frac{U}{\delta^2}\right)$$

i.e. $2\frac{U^2}{L} \sim \nu \frac{U}{\delta^2}$

i.e. $\frac{U^2}{L} \sim \nu \frac{U}{\delta^2}$

i.e. $\delta^2 \sim \nu \frac{L}{U}$

i.e. $\delta \sim \sqrt{\frac{\nu L}{U}}$

Thus $\frac{U^2}{L} \sim \nu \frac{U}{\delta^2}$

i.e. $\delta \sim \sqrt{\frac{\nu L}{U}}$

which shows that $\delta \sim \sqrt{\nu}$.

If R_L' denotes the Reynolds number related to the length of the plate L, then

$$\delta \sim \sqrt{\frac{\nu L}{U}} = \sqrt{\frac{\nu L^2}{UL}} = \sqrt{\frac{\nu}{UL}} \cdot L$$

i.e. $\delta \sim \frac{L}{\sqrt{R_L}}$; $\because R_L = \frac{UL}{\nu}$

Thus we see that as the Reynolds number increases the thickness decreases. The thickness of boundary Layer is defined to be the region where the x-component of velocity u varies from zero to 0·99 U, the upper boundary is fixed arbitrary, it is fixed on percentage (basic).

The definition of boundary Layer is arbitrary because the transition from the velocity in the boundary to that outside, it takes place asymptotically. Since the boundary layer thickness has no practical importance we imply three other types of thickness which are based on physically meaningful measurments.

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