Chinese, Arabian and Japanese Mathematics

(500 BCE–1700 CE)

Introduction

The period from 500 BCE to 1700 CE witnessed remarkable developments in mathematics in China, the Arab and Islamic world, and Japan. These civilizations made significant contributions to arithmetic, algebra, geometry, number theory, and practical computation. Their mathematical achievements emerged from the needs of agriculture, trade, taxation, astronomy, architecture, land measurement, and religious practices.

Chinese mathematicians developed efficient calculation methods, systematic procedures for solving equations, and techniques for measuring geometrical figures. Arab and Islamic mathematicians preserved and expanded the mathematical knowledge of ancient Greece, India, and other civilizations, while establishing algebra as a distinct branch of mathematics and advancing trigonometry and number theory. Japanese mathematicians, particularly during the Edo period (1603–1868), developed distinctive traditions of mathematics known as wasan, including elegant geometrical problems displayed on wooden tablets called sangaku.

These mathematical traditions were not isolated from one another. Knowledge travelled through trade, translation, scholarly exchange, and religious and cultural contacts. Together, they contributed substantially to the development of modern mathematics.

1. Chinese Mathematics

Chinese mathematics developed over many centuries, with important achievements dating back to the first millennium BCE. Unlike the predominantly deductive tradition associated with ancient Greek geometry, Chinese mathematics placed considerable emphasis on practical calculation, algorithms, numerical procedures, and the solution of problems arising in everyday life.

Major developments included the use of counting rods, a decimal place-value system, the compilation of The Nine Chapters on the Mathematical Art, methods for solving simultaneous linear equations, and techniques for extracting square and cube roots.

1.1 The Chinese Number System

One of the most important contributions of ancient China was its decimal place-value system, which enabled efficient calculations with large numbers.

Counting rods and place value: Chinese mathematicians used small rods arranged on a counting surface to represent numbers. Different arrangements of vertical and horizontal rods distinguished digits according to their positions. The positions represented units, tens, hundreds, thousands, and so on.

For example, the number 3,507 can be expressed as

3507= 3 times 1000 + 5 times 100 + 0 times 10 + 7

The position of each digit determined its value. An empty position represented the absence of a value, although a fully developed written zero was not initially used in the same way as in modern notation.

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Decimal notation: The system was based on powers of ten, making it suitable for multiplication, division, extraction of roots, and other numerical operations.

Positive and negative numbers: Chinese mathematicians used counting rods of different colours or orientations to distinguish positive and negative quantities. Red rods commonly represented positive numbers and black rods negative numbers in traditional accounts. These ideas were useful in solving problems involving gains and losses, surpluses and deficits, and systems of equations.

The Chinese abacus: The suanpan, or Chinese abacus, became an important calculating instrument. It enabled rapid arithmetic and remained widely used in commerce and education.

The Chinese system of counting rods was especially important because it allowed mathematicians to manipulate numbers directly and perform calculations that resemble modern written algorithms.

1.2 The Nine Chapters on the Mathematical Art (Jiu Zhang Suan Shu)

The Nine Chapters on the Mathematical Art, known in Chinese as Jiu Zhang Suan Shu, is one of the most influential works in the history of Chinese mathematics. It developed over several centuries, with its principal form generally associated with the Han dynasty, around the first century CE. It contains 246 problems accompanied by computational procedures. The book was not organised as a modern theoretical textbook. Instead, it presented practical problems and explained methods for obtaining their solutions. Its systematic treatment of arithmetic, geometry, and algebra influenced Chinese mathematical scholarship for centuries. The nine chapters are as follows:

Rectangular Fields

Deals with the measurement of rectangular, triangular, trapezoidal, circular, and other areas, including calculations involving land and agricultural fields.

Millet and Rice

Covers the exchange of grains and other commodities, proportional calculations, and problems involving prices and quantities.

Distribution by Proportion

Explains proportional distribution, division of resources, and the allocation of goods or payments.

Short Width

Presents procedures for finding unknown dimensions from geometrical relationships, including the extraction of square and cube roots.

Consultations about Engineering Works

Addresses volumes of solids such as embankments, dikes, canals, and other structures needed for public works.

Fair Taxes

Concerns proportional taxation, transportation, and the distribution of burdens according to quantities and distances.

Excess and Deficiency

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Introduces a method related to double false position, using trial values to solve problems involving unknown quantities.

Rectangular Arrays

Develops procedures for solving simultaneous linear equations using arrays of numbers on counting boards. These methods are related to the principles of modern elimination and determinants.

Right-Angled Triangles

Studies right-angled triangles, unknown lengths, and geometrical problems involving relationships equivalent to the Pythagorean theorem.

The importance of Jiu Zhang Suan Shu lies in its practical and algorithmic approach. It demonstrates that Chinese mathematicians had developed systematic procedures for solving a wide variety of mathematical problems long before the emergence of modern algebraic notation.

1.3 Contributions of Chinese Mathematicians to Algebra

Chinese mathematicians made major advances in algebra, particularly in solving equations, manipulating polynomial expressions, and developing numerical algorithms.

1. **Solving simultaneous linear equations**

Chapter 8 of Jiu Zhang Suan Shu describes methods for solving systems of simultaneous linear equations by arranging coefficients in rectangular arrays on a counting board. The procedure closely resembles the modern method of Gaussian elimination.

For example, consider

$2x + y = 7, \quad x + y = 5$

By subtracting the second equation from the first, we obtain $x = 2$, and hence $y = 3$. Chinese counting-board methods extended such procedures to systems involving several unknowns.

2. **Positive and negative quantities**

Chinese mathematicians used positive and negative numbers in computational procedures. This was particularly important for representing deficits and for eliminating variables in systems of equations.

3. **Polynomial equations**

During the Song and Yuan dynasties (960–1368), Chinese algebra reached a high level of sophistication. Mathematicians developed methods for constructing and solving polynomial equations of high degree.

Important contributors included:

- **Jia Xian (11th century):** Associated with an early form of the binomial coefficient triangle, used in extracting roots and working with powers.
- **Qin Jiushao (1202–1261):** Developed advanced methods for solving polynomial equations and made important contributions to the theory of congruences.
- **Li Ye (1192–1279):** Developed methods for expressing and manipulating polynomial equations using the technique known as tian yuan shu, or the method of the celestial element.
- **Zhu Shijie (c. 1249–1314):** In his work Jade Mirror of the Four Unknowns (1303), extended algebraic methods to equations involving several unknown quantities.

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These developments demonstrate that Chinese algebra was not merely a collection of arithmetic tricks. It included systematic methods for handling equations and polynomial expressions.

1.4 Contributions of Chinese Mathematicians to Geometry

Chinese geometry developed primarily through practical problems involving land surveying, architecture, engineering, astronomy, and the measurement of solids.

Liu Hui (c. 225–295 CE) was one of the most important mathematicians of ancient China. His commentary on Jiu Zhang Suan Shu, written in 263 CE, explained and justified many computational procedures. His contributions included:

- Developing a method of exhaustion to approximate the value of π using polygons inscribed in a circle.
- Improving methods for calculating areas and volumes.
- Explaining geometrical relationships underlying the extraction of square and cube roots.
- Applying geometrical reasoning to practical measurement problems.

Liu Hui obtained an approximation of π close to 3.1416.

Zu Chongzhi (429–500 CE) made a remarkable advance in the calculation of π. He established the famous approximation $355/113 \approx 3.14159292$

which is accurate to six decimal places. This approximation was exceptionally precise for its time.

Other important developments included formulas for the areas of plane figures, volumes of solids, and the use of right-angled triangles in surveying. Chinese mathematicians also developed geometrical procedures for solving problems that could be translated into polynomial equations.

The Chinese tradition therefore combined geometry with numerical computation rather than always treating geometry as a separate theoretical discipline.

2. Arabian and Islamic Mathematics

Between approximately the eighth and seventeenth centuries, the Arab and wider Islamic world became one of the most important centres of mathematical learning. Scholars working in Baghdad, Damascus, Cairo, Córdoba, Samarkand, and other intellectual centres translated, studied, and extended mathematical works from Greek, Indian, Persian, and other traditions.

The term Arabian mathematics is often used in older historical accounts. However, the scholars concerned belonged to diverse ethnic and linguistic backgrounds, and many wrote in Arabic. The expression Arab and Islamic mathematics better reflects this wider intellectual tradition.

The translation movement, including the activities associated with Baghdad's House of Wisdom, helped bring together different mathematical traditions. Scholars did not simply preserve earlier knowledge; they developed new methods in algebra, geometry, arithmetic, trigonometry, and number theory.

2.1 Contributions to Algebra

Algebra was one of the most important achievements of medieval Islamic mathematics.

Al-Khwarizmi (c. 780–850 CE)

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Muhammad ibn Musa al-Khwarizmi is widely regarded as a foundational figure in the history of algebra. His influential work, Al-Kitab al-Mukhtasar fi Hisab al-Jabr wal-Muqabala, presented systematic methods for solving linear and quadratic equations.

The word algebra derives from al-jabr, a term in the title of his treatise. The word algorithm ultimately derives from the Latinised form of his name.

Al-Khwarizmi classified quadratic equations into types and explained how to solve them using operations that correspond to restoration and balancing. Because symbolic algebraic notation was not yet in use in its modern form, he expressed equations verbally and justified solutions through numerical and geometrical reasoning.

For example, a quadratic equation of the form $x^2 + 10x = 39$ can be solved by completing the square:

$x^2 + 10x + 25 = 64$ or $(x + 5)^2 = 8^2$ or $x + 5 = 8$, $- (x + 5) = 8$ or $x = 3, -13$

In the context of the original problem, the positive solution would be selected when x represents a positive physical quantity. Al-Khwarizmi's historical treatment generally considered positive quantities and did not use modern signed-number notation.

His work helped establish algebra as a systematic subject in its own right rather than simply a collection of techniques embedded within arithmetic or geometry.

Other important algebraists

Abu Kamil (c. 850–930) extended the algebraic tradition of al-Khwarizmi and worked with equations involving irrational quantities. His writings influenced later mathematicians, including Fibonacci.

Al-Karaji (c. 953–1029) advanced the algebraic treatment of polynomials and powers. His work helped move algebra toward a more independent study of algebraic operations.

Omar Khayyam (1048–1131) investigated cubic equations and classified them according to their forms. He used intersections of conic sections, such as circles, parabolas, and hyperbolas, to obtain geometrical solutions. His work illustrates the close relationship between algebra and geometry in medieval Islamic mathematics.

Sharaf al-Din al-Tusi (c. 1135–1213) made important contributions to the study of cubic equations, including the investigation of conditions for their positive roots.

Together, these mathematicians extended algebra beyond linear and quadratic equations and laid foundations for later developments in equation theory.

2.2 Contributions to Geometry

Arab and Islamic mathematicians inherited geometrical knowledge from Greek works, particularly Euclid's Elements, and extended it through original investigations.

1. **Thabit ibn Qurra** (826–901)

Thabit ibn Qurra translated and commented on Greek mathematical works. He contributed to geometry, number theory, and the study of geometrical relationships. His work helped preserve and extend the mathematical tradition associated with Euclid and Archimedes.

2. **Abu al-Wafa al-Buzjani** (940–998)

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Abu al-Wafa made important contributions to geometry and trigonometry. He studied geometrical constructions and developed methods useful in astronomy and practical measurement.

3. **Omar Khayyam**

Khayyam's use of conic sections to solve cubic equations was a major example of the interaction between algebra and geometry. Rather than relying exclusively on numerical approximations, he constructed geometrical solutions based on intersections of curves.

4. **Nasir al-Din al-Tusi (**1201–1274)

Al-Tusi made significant contributions to geometry and trigonometry. He helped develop trigonometry as an independent mathematical discipline and wrote systematic treatments of plane and spherical trigonometry. These developments were important for astronomy, navigation, and the study of celestial movements.

5. **Al-Biruni** (973–c. 1050)

Al-Biruni applied mathematical principles to astronomy, geography, and geodesy. He used geometrical and trigonometric methods to investigate the dimensions of the Earth and distances between geographical locations.

The geometrical tradition of the Islamic world thus connected theoretical mathematics with astronomy, geographical measurement, architectural design, and the solution of algebraic equations.

2.3 Contributions to Number Theory

Number theory, the study of the properties of integers, received important contributions from medieval Islamic scholars.

1. **Thabit ibn Qurra and amicable numbers**

Thabit ibn Qurra developed a method for constructing pairs of amicable numbers under suitable arithmetic conditions. Two numbers are called amicable when each is the sum of the proper divisors of the other.

For example, 220 and 284 form an amicable pair:

- The proper divisors of 220 sum to 284.
- The proper divisors of 284 sum to 220.

This relationship illustrates the study of divisor sums, a fundamental topic in number theory.

2. **Al-Karaji and powers of integers**

Al-Karaji investigated the arithmetic of powers and polynomial expressions. His work helped establish methods for manipulating algebraic quantities without always reducing them to geometrical constructions.

3. **Omar Khayyam and the theory of ratios**

Khayyam studied numerical and geometrical relationships, including the theory of ratios and the foundations of numerical quantities. His work formed part of a broader Islamic mathematical engagement with irrational magnitudes and number concepts.

4. **Number systems and arithmetic**

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Islamic mathematicians helped transmit the Hindu-Arabic numeral system and decimal arithmetic to the wider world. Al-Khwarizmi wrote about calculation using Indian numerals, and Latin translations of such works helped make these techniques accessible to European scholars.

5. **Congruences and numerical problems**

Mathematicians in the Islamic world investigated numerical patterns, divisibility, and problems involving integer solutions. Their work formed part of the gradual development of number theory, although the subject would acquire much of its later modern form in subsequent centuries.

The Islamic mathematical tradition was therefore important not only for algebra and geometry but also for arithmetic, divisibility, integer relationships, and the transmission of computational methods.

3. Japanese Mathematics

Japanese mathematics developed under the influence of Chinese mathematical traditions, particularly through the study of Chinese classics such as Jiu Zhang Suan Shu. By the seventeenth century, Japan had developed a distinctive mathematical tradition known as wasan, meaning Japanese mathematics.

The Edo period (1603–1868) was particularly important for the growth of wasan. During this time, mathematical studies flourished among scholars, teachers, merchants, and enthusiastic amateurs. Japanese mathematicians developed new techniques in algebra, geometry, numerical calculation, and the study of curved figures.

Two particularly distinctive features of Japanese mathematics were its advanced algebraic methods and the tradition of presenting geometrical problems on votive tablets called sangaku.

3.1 Contributions of Japanese Mathematicians to Algebra

Japanese algebra developed from Chinese methods of calculation and polynomial manipulation. During the seventeenth century, Japanese mathematicians transformed these inherited techniques into increasingly systematic methods.

1. Yoshida Mitsuyoshi (1598–1672)

Yoshida Mitsuyoshi was an important figure in the popularisation of mathematics in Japan. His book Jinkōki, published in 1627, explained practical arithmetic and calculation using the Japanese abacus, known as the soroban.

The work covered commercial arithmetic, measurements, and problems of everyday life. It helped spread mathematical knowledge beyond a small group of professional scholars and made calculation accessible to merchants and ordinary learners.

2. Seki Takakazu (c. 1642–1708)

Seki Takakazu, also known as Seki Kōwa, is widely regarded as the leading figure in the development of classical Japanese mathematics. He made major advances in algebra, determinants, numerical analysis, and the study of geometrical quantities.

His work transformed algebra from a collection of techniques for solving individual problems into a more systematic mathematical discipline.

His principal contributions included:

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- Advanced algebraic methods: Seki developed techniques for constructing and manipulating polynomial equations arising from geometrical problems.
- Improved algebraic notation: His notation, known as bōshōhō, helped express mathematical relationships more systematically.
- Determinants: He developed determinant methods independently in the Japanese mathematical tradition, around the same period as related European developments. Determinants are now essential in linear algebra and the solution of systems of equations.
- Numerical calculation: He investigated techniques for obtaining accurate numerical values and solving complex equations.
- Geometrical applications: He applied algebraic methods to calculate the areas of figures and the properties of polygons.

Seki's work demonstrates the originality and sophistication of wasan during the seventeenth century.

3.2 Contributions of Japanese Mathematicians to Geometry

Geometry was one of the most creative areas of Japanese mathematics. Japanese scholars investigated circles, polygons, tangencies, geometrical constructions, and relationships between the areas and volumes of figures.

1. Circle geometry

Japanese mathematicians developed methods for solving problems involving circles inscribed in or circumscribed around other geometrical figures. These problems often involved finding unknown radii, distances, or areas from complicated arrangements of circles and polygons.

2. Geometry of polygons

Seki Takakazu investigated the geometrical properties of regular polygons, including methods for calculating their areas and related quantities. He applied algebraic techniques to problems involving regular figures.

3. Areas and volumes

Wasan scholars explored methods for calculating the areas of plane figures and the volumes of solids. Their studies included problems involving curved boundaries and complicated combinations of geometrical shapes.

4. The circle principle (enri or yenri)

Japanese mathematicians developed a tradition of geometrical investigation called enri, or the circle principle. It involved calculating the lengths, areas, and volumes of curved figures through approximation and infinite-series methods.

Some of these investigations resembled techniques associated with integral calculus. They were developed within the Japanese mathematical tradition, independently of many contemporary European approaches, although the precise chronology and scope of the similarities vary by problem.

These geometrical investigations reflected a strong interest in accuracy, elegant construction, and the discovery of general mathematical relationships.

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3.3 Sangaku: Japanese Temple Geometry

One of the most distinctive features of Japanese mathematics was the tradition of sangaku, or mathematical tablets. The word refers to wooden tablets bearing mathematical problems that were dedicated to Shinto shrines and Buddhist temples, particularly during the Edo period.

These tablets often displayed colourful geometrical diagrams and challenging mathematical questions. They were expressions of devotion, demonstrations of mathematical skill, and invitations to other mathematicians to solve or extend the problems.

Main features of sangaku

1. **Geometrical problems**

Many sangaku problems involved circles, triangles, squares, ellipses, and other geometrical figures. A common theme was the relationship between circles touching one another or fitting inside larger figures.

2. **Mathematical challenges**

The tablets frequently presented difficult questions involving unknown lengths, radii, areas, and geometrical relationships. In many cases, the problem and its answer were displayed without a detailed explanation of the solution.

3. **Religious and cultural significance**

The dedication of mathematical tablets to shrines and temples connected mathematics with religious devotion and community life. Mathematical ability was celebrated as a source of beauty, intellectual achievement, and personal dedication.

4. **Public participation in mathematics**

Sangaku encouraged people outside formal scholarly institutions to engage in mathematical thinking. Local schools and groups of enthusiasts developed their own problems and shared their mathematical achievements.

5. **Historical importance**

The surviving tablets provide evidence of the popularity of mathematics in Japan during the Edo period. They also preserve problems that help historians understand the geometrical interests and problem-solving techniques of Japanese mathematicians.

The oldest surviving sangaku is associated with Hoshinomiya Shrine in present-day Tochigi Prefecture and dates to 1683, although the original tablet was damaged by fire in 1975. A duplicate is now displayed. Another early tablet, dating to 1691, is associated with Yasaka Shrine in Kyoto.

Sangaku is important because it illustrates a form of mathematical culture in which problem-solving was not limited to professional scholars or formal classrooms. Mathematics became a public activity, combining intellectual challenge, artistic presentation, and religious expression.

4. Comparative Overview of Chinese, Arabian and Japanese Mathematics

The following table summarises the principal achievements of the three traditions during the period under consideration.

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Field	Chinese mathematics	Arab and Islamic mathematics	Japanese mathematics
Number system	Decimal place value, counting rods, positive and negative numbers	Development and transmission of Hindu-Arabic numerals and decimal arithmetic	Soroban-based calculation and inherited Chinese arithmetic
Algebra	Linear systems, elimination, polynomial equations and numerical root-finding	Systematic algebra, quadratic and cubic equations, polynomial methods	Advanced polynomial algebra, determinants and numerical methods
Geometry	Land measurement, areas, volumes, approximation of \(\pi\)	Euclidean geometry, conic sections, trigonometry and geodesy	Circle geometry, polygon problems, areas, volumes and curved figures
Number theory	Divisibility, remainders and numerical algorithms	Amicable numbers, divisor relationships and arithmetic investigations	Numerical patterns, algebraic computation and problem-solving
Important texts or traditions	Jiu Zhang Suan Shu	Al-Khwarizmi's algebra treatise and later mathematical works	Jinkōki, wasan and sangaku
Notable mathematicians	Liu Hui, Zu Chongzhi, Qin Jiushao, Zhu Shijie	Al-Khwarizmi, Thabit ibn Qurra, Al-Karaji, Omar Khayyam	Yoshida Mitsuyoshi, Seki Takakazu

5. Historical Significance and Influence

The mathematical achievements of China, the Arab and Islamic world, and Japan illustrate the diversity of approaches through which mathematical knowledge developed.

Chinese mathematics demonstrated the power of algorithms and systematic numerical procedures. Its methods for solving simultaneous linear equations, extracting roots, and manipulating polynomial expressions anticipated techniques that later became standard in algebra. The Nine Chapters on the Mathematical Art served as a major textbook for centuries and influenced mathematical education beyond China.

Arab and Islamic mathematics created an important bridge between ancient and later mathematical traditions. Scholars translated and developed Greek and Indian mathematical knowledge, established algebra as a systematic discipline, advanced trigonometry, and contributed to number theory. Their works were translated into Latin and helped shape the development of mathematics in medieval and Renaissance Europe.

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Japanese mathematics demonstrates how an inherited tradition can develop into an original intellectual culture. Through wasan, Japanese mathematicians expanded algebraic techniques and investigated difficult geometrical problems. Sangaku, in particular, reveals that mathematics could flourish through public participation, independent scholarship, artistic expression, and religious dedication.

Although the three traditions differed in their methods and priorities, they shared a commitment to solving mathematical problems through observation, computation, reasoning, and the search for general relationships.

Conclusion

The period from 500 BCE to 1700 CE was a formative era in the global history of mathematics. Chinese mathematicians developed an effective decimal place-value system, sophisticated computational procedures, and advanced methods for solving equations. The Nine Chapters on the Mathematical Art became one of the most influential mathematical texts in East Asia.

Arab and Islamic mathematicians transformed algebra into a systematic discipline, advanced geometrical and trigonometric methods, and made significant contributions to number theory. Their scholarship also played a crucial role in transmitting mathematical knowledge between civilizations.

Japanese mathematicians developed the distinctive tradition of wasan, making original contributions to algebra, determinants, and geometry. The sangaku tradition further demonstrated that mathematics could be a public, cultural, and artistic pursuit.

These achievements remind us that the history of mathematics is not the story of a single civilization. It is the result of contributions from many societies, each developing new methods and ideas in response to its intellectual, cultural, and practical needs. The achievements of China, the Arab and Islamic world, and Japan remain important chapters in the development of modern mathematical thought.

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