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1 (Sem–2/FYUGP) MAT 21 MJ

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MATHEMATICS

(Major)

Paper : MAT4200104MJ

(Calculus)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×8=8

(a) Find $\lim_{x \to 5} \frac{x^2 - 3x - 10}{x^2 - 10x + 25}$.

(b) Define continuity of a function at a point.

(c) Write the n^{th} derivative of $\sin x$.

(d) Write the value of $\int_0^{\pi/2} \sin^8 x \, dx$.

(e) "If two functions $f(x)$ and $g(x)$ have the same derivative for every value of x in $[a, b]$ then they differ only by a constant." *(State True **or** False)*

(f) If $f(x, y) = 2x^3y^2 + 2y + 4x$ then find the value of $f_x(1, 3)$.

(g) If f is a function of two variables then under what condition $f_{xy} = f_{yx}$, for all values of x and y?

(h) If z is a homogeneous function of x, y of order 'n' then write the value of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$.

2. Answer ***any six*** questions : 2×6=12

(a) Explain, why $\lim_{x \to 0} \frac{|x|}{x}$ does not exist.

(b) What can you say about the continuity of the function $f(x) = \sqrt{9 - x^2}$?

(c) If $y = \log(ax + b)$ then find y_n.

(d) Evaluate $\int_0^1 \frac{x^5}{\sqrt{1 - x^2}} dx$.

(e) Give the Geometrical statement of Lagrange's mean value theorem.

(f) Find the first three Taylor polynomials of $\ln x$ about $x = 2$.

(g) Find the Maclaurin series for $\sin x$.

(h) If $f(x, y, z) = \sqrt{1 - x^2 - y^2 - z^2}$ then find $f\left(0, \frac{1}{2}, -\frac{1}{2}\right)$ and the natural domain of 'f'.

(i) If $z = x^4 \sin(xy^3)$ then find $\partial z/\partial x$ and $\partial z/\partial y$.

(j) If $f(x, y) = x^2y + 5y^3$, then find the slope of the surface $z = f(x, y)$ in the x direction at the point $(1, -2)$.

3. Answer ***any four*** questions : 5×4=20

(a) Using ε-δ definition, prove that

$$\lim_{x \to 3} x^2 = 9$$

(b) Find values of the constants k and m, if possible that will make the function 'f' continuous everywhere, where

$$f(x) = \begin{cases} x^2 + 5, & x > 2 \\ m(x+1) + k, & -1 < x \leq 2 \\ 2x^3 + x + 7, & x \leq -1 \end{cases}$$

(c) If $y = a\cos(\log x) + b\sin(\log x)$ then show that

$$x^2 y_{n+2} + (2n+1)\, x y_{n+1} + (n^2+1)\, y_n = 0.$$

(d) Obtain the reduction formula for $\int \cos^n x\, dx$ and hence find the value of

$$\int_0^{\pi/2} \cos^{11} x\, dx.$$

4+1=5

(e) Show that $\frac{x}{1+x} < \log(1+x) < x$ for $x > 0$.

(f) Find the n^{th} Maclaurin polynomial for $\frac{1}{1-x}$ and express it in sigma notation.

4+1=5

(g) Describe the level surfaces of $f(x, y, z) = z^2 - x^2 - y^2$.

(h) If z is a homogeneous function of x, y of degree 'n' then prove that

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = n(n-1)\, z.$$

4. Answer ***any two*** questions : 10×2=20

(a) State and prove Intermediate Value Theorem. Using this, show that if 'f' is continuous on $[a, b]$ and $f(a)$, $f(b)$ are non-zero and have opposite signs then there is *at least one* solution of the equation $f(x) = 0$ in the interval (a, b).

7+3=10

(b) State and prove Leibnitz' theorem. Using Leibnitz' Theorem find the n^{th} derivative of $x^2e^x \cos x$. 7+3=10

(c) Let, $f(x, y) = \begin{cases} -\dfrac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$

(i) Show that $f_x(x, y)$ and $f_y(x, y)$ exist at all points (x, y).

(ii) Show that f is not continuous at $(0, 0)$.

5+5=10

(d) (i) Obtain the reduction formula for $\int \tan^n x \, dx$. Hence evaluate $\int \tan^6 x \, dx$. 3+2=5

(ii) Examine the differentiability of the function

$$f(x) = \begin{cases} \dfrac{xe^{1/x}}{1+e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

at $x = 0$. 5

(e) (i) Find 'c' in the mean value theorem if

$f(x) = (x-1)(x-2)(x-3); a = 0, b = 4$ 5

(ii) Show that between any two roots of $e^x \cos x = 1$ there exists at least one root of $e^x \sin x - 1 = 0$. 5