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**14 (MAT–1) 1056**

**2023**

# MATHEMATICS

Paper : MAT–1056

***(Numerical Analysis)***

*Full Marks : 80*

Time : Three hours

***The figures in the margin indicate full marks for the questions.***

1. Answer ***any four*** parts : 5×4=20

   *(a)* Solve the following system of equations by the LU decomposition method :

$$x_1 + x_2 - x_3 = 2$$
$$2x_1 + 3x_2 + 5x_3 = -3$$
$$3x_1 + 2x_2 - 3x_3 = 6$$

   *(b)* Write briefly about Cholesky method.

*Contd.*

(c) Solve the following system of equations using Cholesky method :

$$4x_1 - x_2 = 1$$
$$-x_1 + 4x_2 - x_3 = 0$$
$$-x_2 + 4x_3 - x_4 = 0$$
$$-x_3 + 4x_4 = 0$$

(d) State and prove Gershgorin's theorem.

(e) Solve the following system of equations :

$$27x_1 + 6x_2 - x_3 = 85$$
$$6x_1 + 15x_2 + 2x_3 = 72$$
$$x_1 + x_2 + 54x_3 = 110$$

using Gauss-Seidel method. Take initial approximation as $X^{(0)} = 0$ and perform three iterations.

2. Answer ***any two*** parts : 5×2=10

(a) Apply Newton-Raphson's method to determine a root of the equation :

$$f(x) = \cos x - xe^x = 0$$

such that $|f(x^*)| < 10^{-8}$, where $x^*$ is the approximation to the root. Take the initial approximation as $x_0 = 1$.



(b) Perform three interations of the Newton-Raphson method to solve the system of equations :

$$x^2 + xy + y^2 = 7$$
$$x^3 + y^3 = 9.$$

Take the initial approximation as $x_0 = 1.5$, $y_0 = 0.5$.

(c) Perform two iterations of the linear iteration method followed by one iteration of the Aitken $\Delta^2$ method to find the root of the equation :

$$f(x) = x^3 - 5x + 1 = 0$$

Take initial approximation as $x_0 = 0.5$. Repeat the process two times.

3. (a) Derive Hermite interpolating polynomial $p(x)$ for a function $f(x)$, if $p(x_i) = f(x_i)$ and $p'(x_i) = f'(x_i)$, $i = 0, 1, 2, \ldots, n$. 10

**Or**

Obtain the cubic spline approximation for the function $f$ defined by the data :

| $x$ | : | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $f(x)$ | : | 1 | 4 | 10 | 8 |

with $M(0) = 0$, $M(3) = 0$. Hence find an estimate of $f(1.5)$.

*(b)* Use the iterative method of inverse interpolation to approximate the value of $x$ (correct to four decimal places) when $f(x) = 0.99800$ from the following table :

| $x$ | : | 1.40 | 1.50 | 1.60 |
|---|---|---|---|---|
| $f(x)$ | : | 0.98545 | 0.99749 | 0.99957 |

| $x$ | : | 1.70 | 1.80 |
|---|---|---|---|
| $f(x)$ | : | 0.99166 | 0.97385 |

5



4. *(a)* Derive the formula for

$\int_a^b f(x)\,dx = \lambda_0 f(a) + \lambda_1 f(b)$, if this formula gives exact value of $\int_a^b (a_0 x + b_0 x^2)\,dx$, i.e, a linear combination of $x$ and $x^2$.

Evaluate $\int_2^5 (2x^2 - 3x)\,dx$ using the above rule. 4+1=5

**Or**

Derive Gauss-Legendre three point integration method with the help of Legendre polynomial $p_3(x)$. 5

*(b)* Derive Stirling's approximation to $\lfloor n$ using Euler-Maclaurin's formula. 10

**Or**

Find an approximate value of

$$\int_0^1 \left(1+\frac{\sin x}{x}\right) dx$$

using composite trapezoidal rule with 2, 3, 5 nodes and Romberg integration. Assume $f(0)$ is taken as the limiting value.

5. Answer ***any two*** parts : 10×2=20

(a) Using modified Euler's method, obtain a solution of the equation :

$$\frac{dy}{dx} = x + \left|\sqrt{y}\right|$$

with initial condition $y = 1$ at $x = 0$ for the range $0 \le x \le 0.6$ in steps of 0.2.

(b) If $y(x)$ is the solution of

$$\frac{dy}{dx} = \frac{1}{2}(x+y),\ y(0) = 2,$$

estimate $y(2)$ using Milne-Simpson predictor-corrector method. Take $h = 0.5$.



(c) Compute $y(0.8)$ by Adams-Moulton method from

$$\frac{dy}{dx} = 1 + y^2,\ y(0) = 0,$$

taking the step length $h = 0.2$.

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