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14 (MAT–2) 2056(O)

2023

MATHEMATICS

Paper : MAT–2056

(Old Course)

(Mathematical Methods)

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Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any three*** parts : 5×3=15

   (a) Transform the boundary value problem $y''(x)+y(x)=0;\ y(0)=1, y'(1)=0$ into an integral equation.

   (b) Using the method of successive approximation, solve the integral equation

$$\phi(x)=\frac{5x}{6}+\frac{1}{2}\int_0^1 xt\,\phi(t)\,dt$$

Contd.

(c) Find the resolvent kernel of the integral equation

$$\phi(x) = e^x - \frac{1}{2}e + \frac{1}{2} + \frac{1}{2}\int_0^1 \phi(t)\,dt$$

and hence solve it.

(d) Find the eigenvalues and eigenfunctions for the integral equation

$$\phi(x) = \lambda \int_0^{\pi} \cos(x+t)\,\phi(t)\,dt$$

2. Answer ***any two*** parts : 5×2=10

(a) Form an integral equation corresponding to the differential equation

$$\frac{d^3y}{dx^3} + x\frac{d^2y}{dx^2} + (x^2 - x)y = xe^x + 1$$

with initial conditions

$y(0) = 1 = y'(0),\ y''(0) = 0.$

(b) Find the resolvent kernel of the Volterra integral equation with the kernel

$$k(x, t) = \frac{2 + \cos x}{2 + \cos t}.$$

(c) Using the method of successive approximation, solve the integral equation $\phi(x) = 1 + \int_0^x \phi(t)\,dt$, taking

$\phi_0(x) = 0.$

3. Answer ***any three*** parts : 5×3=15

(a) Find $F_S\{F(x)\}$, where

$$F(x) = \begin{cases} \sin x, & 0 < x < a \\ 0, & x > a \end{cases}$$

(b) Show that $\int_0^{\infty} \frac{\cos \lambda x}{\lambda^2 + 1}\,d\lambda = \frac{\pi}{2}e^{-x},\ x > 0.$

(c) Using the Fourier sine transform, solve the partial differential equation

$$\frac{\partial U}{\partial t} = 2\frac{\partial^2 U}{\partial x^2} \text{ if } U(0,t) = 0,$$

$U(x,0) = e^{-x}$, $x > 0$ and $U(x,t)$ is bounded for $x > 0, t > 0$.

(d) Find $F_S^{-1}\left\{\frac{e^{-a\lambda}}{\lambda}\right\}$.

4. Answer ***any three*** parts : 5×3=15

(a) If F is a periodic function of period $T(T > 0)$, then show that

$$L\{F(t)\} = \frac{\int_0^T e^{-ST} F(t)\, dt}{1 - e^{-ST}}.$$

(b) Prove that $\int_0^\infty \frac{\cos at - \cos bt}{t}\, dt = \log\frac{b}{a}$.

(c) Use convolution theorem, to evaluate

$$L^{-1}\left\{\frac{1}{s(s^2+4)}\right\}.$$

(d) Using Laplace transform technique, solve the differential equation

$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 0$ subject to the conditions $x(0) = 0, x'(0) = 1$.

5. Answer ***any two*** parts : 5×2=10

(a) Find the extremal of the functional

$$I[y(x)] = \int_1^2 \frac{\sqrt{1+y'^2}}{x}\, dx;\ y(1) = 0,$$

$y(2) = 1$

(b) Find the curve with fixed boundary points such that its rotation about the axis of abscissae give rise to a surface of revolution of minimum area.

(c) Find the stationary value or extremal of the functional $\int_0^{\pi/2} \left(y'^2 - y^2 + 2xy\right) dx$

with $y(0) = 0$ and $y\left(\frac{\pi}{2}\right) = 0$.

6. (a) Find the extremal for the functional $I[x(t), y(t)]$

$$= \int_{t_1}^{t_2} \left[\left\{ \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 \right\}^{1/2} + a^2 \left(x\frac{dy}{dt} - y\frac{dx}{dt} \right) \right] dt,$$

where a being a constant. 5

(b) Find a function $y(x)$ for which $\int_0^1 \left(x^2 - y'^2\right) dx$ is stationary, given that

$\int_0^1 y^2 dx = 2$, $y(0) = 0$, $y(1) = 0$. 5

(c) What is an isoperimetric problem ? How can one convert an isoperimetric problem to the variational problem under subsidiary conditions ? 5

OR

Determine the extremal of the functional

$$I[y(x)] = \int_{-L}^{L} \left(\frac{1}{2} \mu y''^2 + \lambda y \right) dx,$$

subject to the conditions

$y(-l) = 0$, $y'(-l) = 0$, $y(l) = 0$, $y'(l) = 0$.