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14 (MAT–4) 4016 (N/O)

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MATHEMATICS

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Estd.-1964
Boko, Dist - Kamrup

Paper : MAT–4016

(New/Old Course)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

(Symbols have their usual meaning)

(Fuzzy Sets and Systems)

1. Write a short note on the importance of fuzzy set and fuzzy logic from the perspective of Professor Lofti Zadeh. (within **500** words) 5

2. Answer ***any three*** parts : 5×3=15

   (a) State extension principle for fuzzy sets. Let $f : X \to Y$ be an arbitrary crisp function. Then for any $A \in \mathcal{F}(X)$, prove that f fuzzyfied by the extension principle satisfies the equation. 2+3=5

$$f(A) = \bigcup_{\alpha \in [0,1]} f(_{\alpha+}A)$$

Contd.

(b) Write the axioms of fuzzy complement function. Prove that every fuzzy complement has atmost one equilibrium. 2+3=5

(c) Define t-conorms. Prove that the standard fuzzy union is the only idempotent t-norm. 2+3=5

(d) Let A and B be *two* triangular shaped fuzzy numbers defined as follows :

$$A(x)=\begin{cases}0 & ; \quad x<-2 \text{ and } x>4\\ \frac{x+2}{3} & ; \quad -2\le x\le 1\\ \frac{4-x}{3} & ; \quad 1\le x\le 4\end{cases}$$

and

$$B(x)=\begin{cases}0 & ; \quad x<1 \text{ and } x>3\\ x-1 & ; \quad 1\le x\le 2\\ 3-x & ; \quad 2\le x\le 3\end{cases}$$

Then, find $(A-B)(x)$.

3. Answer ***any two*** parts : 5×2=10

(a) Define first and second projections of a fuzzy relation. Let $R(X,Y)$ be a fuzzy relation defined by the following relation matrix :

	y_1	y_2	y_3	y_4	y_5
x_1	0.8	0	0.3	0.4	0
x_2	0	1	0.5	0	0.5
x_3	1	0.4	0.6	0.8	0.6

Then, find the first, second and total projections. Write cylindrical extensions of $R^{(1)}$ and $R^{(2)}$ 1+2+2=5

(b) The fuzzy binary relation R is defined on sets $X=\{1,2,3,\cdots,100\}$, $Y=\{50,51,\cdots,100\}$ and represents the relation 'x is much smaller than y'. It is defined by membership function

$$R(x,y)=\begin{cases}1-\frac{x}{y} & ; x\le y\\ 0 & ; \text{otherwise}\end{cases}$$

Here $(x,y)\in X\times Y$. Then,

(i) find the domain of R;

(ii) find the range of R;

(iii) find the length of R;

(iv) calculate R^{-1}. 1+1+1+2=5

(c) Let $P=\begin{pmatrix}0.3 & 0.5 & 0.8\\ 0 & 0.7 & 1\\ 0.4 & 0.6 & 0.5\end{pmatrix}$

and

$$Q=\begin{pmatrix}0.9 & 0.5 & 0.7 & 0.7\\ 0.3 & 0.2 & 0 & 0.9\\ 1 & 0 & 0.5 & 0.5\end{pmatrix}$$

Then, find $R=P\circ Q$.

(d) Write a brief note on fuzzy compatibility relations.

(e) Write a short note on the following sentence :

"$\prod(\alpha_R)$ is refinement of $\prod(\beta_R)$ if and only if $\alpha \geq \beta$"

4. Answer ***any two*** parts : 5×2=10

(a) Define belief measure and plausibility measure. Prove that $Bel(B) \geq Bel(A)$ if $A \subseteq B$ and $A, B \in P(X)$. 1+1+3=5

(b) State the differences between probability distribution functions and basic probability assignments.

(c) Let $X = \{a, b, c, d\}$. Given the belief measure $Bel(\{b\}) = 0.1$, $Bel(\{a,b\}) = 0.2$, $Bel(\{b,c\}) = 0.3$ and $Bel(\{b, d\}) = 1$. Determine the corresponding basic assignment.

(d) Prove that $Pl(A) \geq Bel(A)$, for all $A \in P(X)$.

(e) Discuss basic assignment on the Cartesian product of X and Y.

5. Answer ***any two*** parts : 5×2=10

(a) Define logic function, logic formula and logic primitive. Give two different examples of logic formulas which represent the same logic function. (1+1+1)+2=5

(b) Using Lukasiewicz's three-valued logic, find the truth value of the following : $(\sim a \wedge b) \Rightarrow b$, where a, b are logic variables and $\sim a = 1 - a$.

(c) Let $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2\}$ be two crisp sets respectively. Assume that the proposition "if x is A, then y is B" is given, where $A = \frac{0.5}{x_1} + \frac{1}{x_2} + \frac{0.6}{x_3}$ and $B = \frac{1}{y_1} + \frac{0.4}{y_2}$. Then given a fact expressed by the proposition "x is A'", where $A' = \frac{0.6}{x_1} + \frac{0.9}{x_2} + \frac{0.7}{x_3}$. Give the conclusion in the form "y is B'". Hence, find the truth value of the proposition "y_1 is B' is very true", where the corresponding modifier is suitably defined by you. 4+1=5

6. Answer ***any two*** parts : 5×2=10

(a) Analyse the truth value of the fuzzy proposition "There are about three students in the class who are tall and young" with suitable fuzzy and crisp sets defined by you.

(b) Show that Lukasiewicz implication is both S-implication and R-implication.

(c) Let i and u be the standard fuzzy intersection and fuzzy union respectively. Let J be the fuzzy implication that satisfies.

(a) $a \le b$ implies $J(a, x) \ge J(b, x)$,

(b) $a \le b$ implies $J(x, a) \le J(x, b)$.

Prove that $J(a, i(b, c)) = i(J(a, b), J(a, c))$ and $J(a, u(b, c)) = u(J(a, b), J(a, c))$

7. Answer ***any two*** parts : 5×2=10

(a) Write a short note on individual decision making in fuzzy environment.

(b) Assume that each individual of a group of five judges has a total preference ordering P_i on four sports persons a, b, c, d. The orderings are —

$P_1 = (a, b, d, c)$, $P_2 = (a, c, d, b)$,

$P_3 = (b, a, c, d) = P_5$, $P_4 = (a, d, b, c)$.

Use fuzzy multi person decision making to determine the group decision.

(c) Write a short note on fuzzification process of a fuzzy controller.

8. Answer ***any one*** part : 10

(a) Solve the following fuzzy linear programming problem :

$$Max\ Z = 0.5x_1 + 0.2x_2$$

such that $x_1 + x_2 \le B_1$,

$2x_1 + x_2 \le B_2$,

$x_1, x_2 \ge 0$

where, $B_1(x) = \begin{cases} 1 & ; \ x \le 300 \\ \frac{400-x}{100} & ; \ 300 < x \le 400 \\ 0 & ; \ x > 400 \end{cases}$

and $B_2(x) = \begin{cases} 1 & ; \ x \le 400 \\ \frac{500-x}{100} & ; \ 400 < x \le 500 \\ 0 & ; \ x > 500 \end{cases}$

(b) Write a short note on fuzzy systems and neural networks.

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(Space Dynamics)

1. (a) Show that the centre of mass of the two body problem moves uniformly in a straight line in space. 5

   (b) Show that the solution of the two body problem represent a conic. 5

2. Answer **any two** parts : 5×2=10

   (a) Show that the velocity of planet moving in an ellipse about a centre of force in the focus is compounded of two constant velocities μ/h perpendicular to the radius and $\mu e/h$ perpendicular to the major axis.

   (b) Communication satellites are launched in circular orbit in equatorial plane of the earth so that they remain stationary relative to the surface of the earth. What is the minimum number of satellites required so that every place may view at least one satellite ?

   (Given GM = 398603·6 km^3/s^2)

   (c) A satellite has its largest and smallest orbital speeds given by v_{max} and v_{min} respectively. If T is the period of revolution of the satellite, show that the semi-major axis of the orbit is

$$\frac{1}{2\pi}\sqrt{v_{max}\cdot v_{min}}$$

3. Answer **any two** parts : 10×2=20

   (a) Prove the expression for Kepler's problem

$$E = m + \left(e - \frac{e^3}{8}\right)\sin m + \frac{e^2}{2}\sin 2m + \frac{3}{8}e^3 \sin 3m$$

   (b) What is the meaning of three body problem ? Show that the equations of motion in restricted three body problem are

$$\ddot{x} - 2\dot{y} = x - \left(\frac{1-m}{\rho_1^3}\right)(x - x_1) - \frac{m}{\rho_2^3}(x - x_2),$$

$$\ddot{y} - 2\dot{x} = y - \left(\frac{1-m}{\rho_1^3}\right)y - \frac{m}{\rho_2^3}y,$$

$$\ddot{z} = -\frac{(1-m)}{\rho_1^3}z - \frac{m}{\rho_2^3}z$$

   where the symbols have their usual meanings.

(c) Derive the integral of energy of the three body problem.

4. Answer **any two** parts : 10×2=20

(a) (i) Write a short note on orbital elements with proper diagram. 6

(ii) Show that under no perturbing forces if v_1 and v_2 represents the linear velocities of a planet when it is respectively nearest and farthest from the sun, then

$$(1-e)v_1 = (1+e)v_2$$ 4

(b) (i) Write a short note on the classifications and components of a perturbing force. 4

(ii) Derive the components of the perturbing force due to the action of the sun on the moon moving around the earth. 6

(c) (i) Show that a positive tangential component does not change the eccentricity when the moving planet is at any end of the minor axis. 5

(ii) Derive the differential equation of motion of an orbit under a small atmospheric resistance that is proportional to orbital velocity and inversely proportional to the square of the distance between the centre of masses. 5

5. Answer **any two** parts : 10×2=20

(a) (i) A space vehicle is moving in an elliptic orbit of period T under the attraction of the sun of mass M. The motors are fired momentarily so that its orbital speed v is suddenly increased by the amount Δv. Show that the resulting change ΔT in period is given by

$$\Delta T = 3(2\pi GM)^{-2/3} T^{5/3} v \Delta v,$$

G is the constant of gravitation. 6

(ii) Discuss the advantage of a multistage rocket over a single stage rocket. 4

(b) (i) What is a bi-elliptic Hohmann transfer and what is its advantage in comparison to regular Hohmann transfer ? 2+1=3

(ii) A satellite of mass m_s is in an elliptical orbit around a planet of mass $m_p >> m_s$. The planet is located in one focus of the ellipse. The satellite is at the distance r_a when it is farthest from the planet. The distance of closest approach is r_p. What is the speed v_p of the satellite when it is closest to the planet and the speed v_a of the satellite when it is farthest from the planet? 5

(iii) What effect is produced in the velocity increment of a single stage rocket operating in a gravity free space on doubling the exhaust velocity? 2

(c) Explain the dynamics of a three stage rocket under constant gravitational field. 10
