Total number of printed pages–11

14 (MAT–4) 4036 (N/O)

2023

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MATHEMATICS

Paper : MAT 4036

(New/Old Course)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

(Advanced Algebra)

1. Answer ***any three*** parts : 5×3=15

   (a) Define self dual poset. Show that the poset $(P(X),\subseteq)$ of all subsets of a non-empty set X is self dual. 1+4=5

   (b) Show that the cardinal product of *two* self dual posets is self dual.

   (c) Define a distributive Lattice. Show that a distributive Lattice is always modular but the converse may not be true.

Contd.

(d) If $\{G_i / i \in I\}$ is a family of groups, then show that the direct product $\prod_{i \in I} G_i$ is a group.

2. Answer ***any two*** parts : 5×2=10

(a) State Sylow's second theorem. Show that if there is only one Sylow p-subgroup of a group G, then it is a normal subgroup of G. 1+4=5

(b) Show that a group of order p^m possesses at least one normal subgroup of order p^i, where $1 \leq i \leq m$.

(c) There is no simple group of order 200. Justify.

3. Answer ***any three*** parts : 5×3=15

(a) Show the equivalence of the following statements of an abelian group F :

(i) F has a non-empty basis.

(ii) F is the (internal) direct sum of a family of infinite cyclic subgroups.

(iii) F is (isomorphic to) a direct sum of copies of the additive group of integers.

(b) Define rank of a free abelian group. Show that every abelian group G is the homomorphic image of a free abelian group of rank $|X|$, where X is a set of generator of G.

(c) Let F_1 and F_2 be the free abelian groups on the sets X_1 and X_2 respectively. Show that $F_1 \cong F_2$ if and only if F_1 and F_2 have same rank, i.e., $|X_1| = |X_2|$.

(d) Show that every free group is torsion free.

4. Answer ***any four*** parts : 5×4=20

(a) Define a right R-module and establish the following :

Let $\Gamma : R \to F$ be a homorphism of the ring R into the ring F of right endomorphisms of the additive abelian group A then by putting $ar = a\Gamma r$ for all $a \in A, r \in R$ a right R-module A_R is obtained and every right R-module may be obtained in this way. 1+4=5

(b) Show that a proper ideal P of a commutative ring R is a prime ideal if and only if for $a, b \in R, ab \in P$ implies $a \in P$ or $b \in P$. Hence show that P is a prime ideal of R if and only if R/P is an integral domain. 3+2=5

(c) Define radical of R. Show that radical of R consists of all elements $r \in R$ such that $1-rx$ is a unit for all $x \in R$. 1+4=5

(d) Let R be a commutative ring with unity. If T is a subset of R which is closed under finite products and does not contain zero, then show that any ideal maximal in the set of ideals not meeting T is a prime ideal of R.

(e) Let R be a commutative ring with unity. Then prove that the following conditions are equivalent :

(i) Every irreducible fraction has domain R.

(ii) For every fraction f in R there exists an element $s \in R$ such that $f(d) = sd$, for all $d \in D$, the domain of f.

(iii) $Q(R) \cong R$, where $Q(R)$ is the complete ring of quotients of R.

5. Answer ***any four*** parts : 5×4=20

(a) Define an irreducible module. If A_R is an irreducible module, show that the set of endomorphisms of A forms a division ring.

(b) If M is a maximal right ideal of ring R, show that $R : M = \{r \in R / Rr \subseteq M\}$ is a primitive ideal of R.

(c) If R denotes a ring, prove that R is a primitive ring if and only if there exists a faithful irreducible module A_R.

(d) Prove that the radical of a right Artinian ring is nilpotent. Hence deduce that in a right Artinian ring rad R = Rad R.

(e) Define a Noetherian ring. Let R be a ring and I be an ideal of R. If I and R/I both are right Noetherian rings, show that R is also a right Noetherian ring.

(Fluid Dynamics)

1. Answer ***any two*** parts : 10×2=20

(a) In the case of surface waves, show that the condition to be satisfied at the free surface by velocity potential is $\frac{\partial^2\phi}{\partial t^2}+g\frac{\partial\phi}{\partial y}=0$ and $\dot{n}=-\frac{\partial\phi}{\partial y}$, where the symbols have their usual meanings.

(b) Find the total energy of progressive waves.

(c) (i) Show that the complex potential for a progressive wave in a deep canal is $w=ace^{-i(mz-nt)}$, where the symbols have their usual meanings. 5

(ii) Write a short note on group velocity. 5

2. Answer ***any two*** parts : 10×2=20

(a) Show that the equation of motion of an incompressible viscous fluid flow under conservative force is

$$\frac{\partial\vec{q}}{\partial t}-\vec{q}\times\vec{\xi}=-\vec{\nabla}\left(v+\frac{p}{\rho}+\frac{1}{2}q^2\right)-\nu\,\vec{\nabla}\times\vec{\xi}.$$

(b) Prove that in the slow steady motion of a viscous fluid in *two* dimensions

$\nu \nabla^4 \psi = \frac{\partial X}{\partial y} - \frac{\partial Y}{\partial x}$, where (X, Y) is the impressed force per unit mass.

(c) Show that the vorticity vector $\vec{\xi}$ of an incompressible viscous fluid moving under conservative forces satisfies the differential equation

$$\frac{d\vec{\xi}}{dt} = (\vec{\xi}.\vec{\nabla})\vec{q} + \nu \nabla^2 \vec{\xi}.$$

3. Answer ***any two*** parts : 10×2=20

(a) With usual notations, obtain the fluid velocity distribution in case of a plane Poiseuille flow in the form

$u = \frac{Ph^2}{8\mu}\left[1 - 4\left(\frac{y}{n}\right)^2\right]$. Hence show that

$$\frac{u_{max}}{u_{average}} = \frac{3}{2}$$

(b) In a steady motion of a viscous liquid through a cylindrical pipe, show that the equation of motion becomes

$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = -\frac{P}{\mu}$, where μ is the viscosity, w and $-P$ are respectively the velocity and pressure gradient along the axis of the cylinder. Further, show that in the case of elliptic cylinder of cross section given by

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the flux of the liquid through the cylinder is $Q = \frac{\pi}{4\mu}\frac{Pa^3b^3}{a^2 + b^2}$. Also obtain the maximum velocity.

(c) The space above the plane $y = 0$ is filled with a liquid of kinematic viscosity ν. Initially the plane and the liquid are at rest. At time $t = 0^+$, the plane suddenly begins to move parallel to itself with a constant velocity U and the liquid also moves in the same direction. Show that the solution of the flow problem is given by

$u = U\left[1 - \frac{2}{\sqrt{\pi}}\int_0^{\eta} e^{-n^2}\, d\eta\right]$, where $\eta = \frac{y}{2\sqrt{\nu t}}$.

(d) Show that for a steady axi-symmetric flow at small Reynolds number, Stokes' stream function ψ satisfies the equation $E^4(\psi) = 0$, where

$$E^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{Sin\theta}\frac{\partial}{\partial\theta}\right).$$

4. Answer ***any two*** parts : 10×2=20

(a) Obtain the Blasius solution in the form

$$f(\eta) = \frac{n^2}{\lfloor 2}\alpha - \frac{n^5}{\lfloor 5}\cdot\frac{\alpha^2}{2} + \frac{\eta^8}{\lfloor 8}\cdot\frac{11\alpha^3}{2^2} - \ldots.$$

for small η, where $\alpha = f''(0)$.

(b) Derive Karman's momentum integral equation. Also, deduce that for steady flow and under no pressure gradient it reduces to

$$\tau_0 = \rho\frac{\partial}{\partial x}\int_0^\infty u(U-u)dy \text{ , where}$$

$$\tau_0 = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \rho \text{ is the density, } u \text{ the}$$

velocity, U the free stream velocity, μ is the coefficient of viscosity.

(c) Show that for a flat plate of length L placed lengthwise in a uniform stream, the assumption $u = \frac{Uy}{\delta^2}(2\delta - y)$ in Karman's integral condition, leads to

$$\delta = \sqrt{\frac{30\nu x}{U}}$$

with a frictional resistance $8\sqrt{\frac{\mu U^3 1\rho}{30}}$, where U is the velocity in the uniform stream.

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